Roll No. **Total No. of Pages : 04**

Total No. of Questions : 09

B.Tech. (Batches 2005-2010) (Sem.–2nd)

ENGG. MATHEMATICS-II

Subject Code : AM-102

Paper ID : [A0119]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Answer briefly :

(a) Test whether the set of vectors

$\{(1, 1, 1), (1, -1, 1), (3, -1, 3)\}$

is linearly dependent or independent.

(b) Find the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & -1 & 2 \\ 0 & 2 & 1 & -1 \end{pmatrix}$$

(c) Prove that eigen values of the Hermetian matrix are purely real.

(d) Find the integrating factor of the equation :

$(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

(e) Find the complete solution of the equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0,\ x(0) = 0,\ \frac{dx}{dt} \text{ at } t = 0 = 15$$

(f) Give physical interpretation of 'curl' of a vector point function.

(g) Find the 'Flux' of $\vec{F} = (x - y)i + xj$ across the circle $x^2 + y^2 = 1$ in the x-y plane.

(h) A continuous r.v. X has a p.d.f. $f(x) = 3x^2$, $0 \le x \le 1$. Find b s.t. $P[X > b] = 0.05$

(i) Write any four chief characteristics of Normal probability curve.

(j) Show that the fluid motion given by

$$\vec{v} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$$

is irrotational.

SECTION-B

2. (a) Use Gauss-Jordan row-reduction method to find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

(b) Test the consistency of the system of equations :

$x + 2y - z = 3;\ 3x - y + 2z = 1;\ 2x - 2y + 3z = 2;\ x - y + z = -1.$

If consistent solve it completely. (3, 5)

3. (a) Solve the differential equation

$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)\ dy = 0$

(b) Solve : $x - \tan^{-1}p = \dfrac{p}{1+p^2}$, where $p = \dfrac{dy}{dx}$ (4, 4)

4. (a) Use method of variation of parameters to solve

$y'' + 3y' + 2y = 2e^x.$

(b) Apply operator method to solve the simultaneous differential equations :

$$\frac{dx}{dt} + x - 3y = 0; \frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = \sin t \,. \qquad (4, 4)$$

5. An LCR circuit with battery e.m.f. E sin pt is tuned to resonance so that $p^2 = \frac{1}{LC}$. Show that for small values of $\frac{R}{L}$, the current in the circuit at any time t is $\frac{Et}{2L}$ sin pt. (8)

SECTION-C

6. (a) Prove that

$$\nabla . (\vec{F} \times \vec{G}) = \vec{G} \cdot (\nabla \times \vec{F}) - \vec{F} \cdot (\nabla \times \vec{G})$$

(b) Find the work done by the force

$$\vec{F} = (2y + 3)\hat{i} + xz\,\hat{j} + (yz - x)\,\hat{k}$$

When it moves a particle from the point (0, 0, 0) to the point (2, 1, 1) along the curve $x = 2t^2$, $y = t$ and $z = t^3$. (4, 4)

7. (a) State Green's theorem in plane and use it to evaluate the integral

$$\oint_C 3y\,dx + 2x\,dy$$

where C : the boundary of $0 \leq x \leq \pi$, $0 \leq y \leq \sin x$

(b) Using Gauss divergence theorem evaluate

$$\int_C \vec{F} \cdot \hat{n}\,ds,$$

where $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$ over the surface S of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. (4, 4)

8. (a) An insurance company insures 4000 people against loss of both eyes in a car accident. Based on previous data, the rates were computed on the assumption that on the average 10 persons in 1,00,000 will have car accident each year that result in this type of injury. What is the probability that more than 3 of the insured will collect on their policy in a given year ?

(b) Fit a least square geometric curve $y = ax^b$ to the data :

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(4, 4)

9. (a) The following random samples are measurements of heat producing capacity in thousands of calories per ton of specimens of coal from two mines :

Mine I	8,260	8,130	8,350	8,070	8,340	
Mine II	7,950	7,890	7,900	8,140	7,920	7,840

Test at 5% level of significance whether the difference between the means of these two samples is significant.

Given $t_{9,\ 0.05} = 2.26$, $t_{10,\ 0.05} = 2.23$, $t_{11,\ 0.05} = 2.20$.

(b) There are two different choices to stimulate a certain chemical process. To test whether the variance of the yield is the same no matter which catalyst is used, a sample of 10 batches is produced using the first catalyst and of 12 using the second. If the resulting data is $S_1^2 = 0.14$, $S_2^2 = 0.28$, then test the hypothesis of equal variance at 2% level. Given that $F_{11,\ 9,\ 0.02} = 5.20$, $F_{9,\ 11,\ 0.02} = 2.95$. (4, 4)